Relaxation cascade in solids: microscopic theory

Yu.N. Ovchinnikov[1,2] and V.Z. Kresin[1]

[1] Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA
[2] Landau Institute for Theoretical Physics, Russian Academy of Sciences, Moscow 11733, Russia

Abstract. The relaxation of an electron excited to the high energy region, is accompanied by the creation of various excitations (plasmons, quasi-particles, phonons). The stages of this many-body, non-stationary phenomenon (cascade) are described microscopically. The electron distribution function $n(\varepsilon, t)$ and characteristic times for the whole energy range, are calculated.

1 Introduction

This paper is concerned with the relaxation phenomenon in solids. Non-equilibrium state could be created by incoming radiation (*e.g.*, by X or γ rays), or by incoming electron beams. The energy input leads to an appearance of quasiparticles whose initial energy ε_i usually greatly exceeds the Debay energy ($\varepsilon_i \gg \tilde{\Omega}$, $\tilde{\Omega} \cong \Omega_D$). The following relaxation process is a non-stationary phenomenon which is accompanied by many collisions and creation new excitations. The analysis of this process, so-called a cascade, is interesting for its own sake, but it is also important for making various detectors (see *e.g.* [1,2]).

As is known, the cascade contains several stages [3,4]. The initial stage corresponds to the high energy region ε_i, so that the relaxation proceeds through electronic channels. If $\varepsilon_i > \omega_p$ (ω_p is the plasmon energy), then at first one can observe a creation of collective electronic excitations (plasmons) which follows by plasmons decay and by electron-electron collisions. The consequent lowering of quasiparticle energy leads to the situation when the scattering by optical and acoustic phonons becomes dominant. We described in our paper [5] the last stage of the cascade, namely, the relaxation caused by scattering of carriers by acoustic phonons. We analyzed also the tunneling superconducting detectors. In the present paper we consider the whole energy range and various relaxation channels.

The cascade has been analyzed in many interesting papers (see, *e.g.* [6–11]). A phenomenological model used in many studies, was introduced in [3]. Microscopic treatment for the case close to equilibrium was described in [6]. A phenomenological approach based on the two-temperature model (TTM, see *e.g.* [7–10]) has been also popular. Nevertheless, it is known (see, *e.g.* [11]), that the TTM model is unable to provide a description in some real cases.

Our goal is to develop a microscopic theory of the cascade. The cascade is a strong non-stationary phenomenon, and we consider various relaxation channels.

We focus on the relaxation in metals and semiconductors with high carrier concentration, so that the value of the plasmon energy greatly exceeds the phonon energy scale ($\omega_p \gg \Omega$). Some systems (*e.g.*, 2D electron gas, layered conductors, semiconductors with low carrier concentration) are characterized by presence of low energy plasmon modes; they will be considered elsewhere.

The structure of the paper is as follows. Section 2 describes the electronic stages of the cascade; they involve collective and one-particle excitations. Relaxation *via* scattering by phonons is analyzed in Section 3. Section 4 contains a general discussion and summary of results.

2 Cascade. Electronic channels

Electronic losses and plasmons

Consider initially excited electronic state. This state may occur through excitation by high energy quanta (*e.g.*, X-rays) or because of injection of electrons by electronic beams. As a result, we create a non-equilibrium distribution $n(\varepsilon, t)$, so that at the beginning of the cascade the system contains excited carriers; additional energy of such carrier $\varepsilon - \varepsilon_F$ greatly exceeds that for phonons. Then the relaxation occurs initially through electronic channels. If the value of the initial energy ε_i is such that $\varepsilon_i > \omega_p$, ω_p is the plasmon energy, then the creation of plasmons is

a first stage of the cascade. Usual treatment of such phenomenon contains calculation of the energy losses (see, e.g., [12–14]). Our goal is different. We are interested in the evolution of the distribution function $n(\varepsilon, t)$ and in the corresponding time scale. According to [12–14] one can write the following set of equations:

$$\partial n(\varepsilon, t)/\partial t = -\lambda_{pl} \left(\omega_p^3/\varepsilon\right)^{1/2} n(\varepsilon, t)\theta(\varepsilon - \omega_p)$$
$$+ \lambda_{pl} \left(\omega_p^3/(\varepsilon + \omega_p)\right)^{1/2} n(\varepsilon + \omega_p) \quad (1a)$$

$$\partial N_{pl}/\partial t = \lambda_{pl} \int_{\omega_p}^{\infty} d\varepsilon n(\varepsilon, t) \left(\omega_p^3/\varepsilon\right)^{1/2} - N_{pl}/\tau_{pl}. \quad (1b)$$

Here N_{pl} is the plasmon distribution function, τ_{pl} is the characteristic time for the plasmon decay into electron-hole pairs, λ_{pl} is the electron-plasmon coupling constant. The coefficients in equation (1a) are written with the logarithmic accuracy.

Assume that $\varepsilon_i \gg \omega_p$, then equation (1a) can be reduced to the form:

$$\partial n(\varepsilon, t)/\partial t = \lambda_{pl}\omega_p(\omega_p/\varepsilon)^{3/2}(-n(\varepsilon, t)/2 + \varepsilon\partial n(\varepsilon, t)/\partial t). \quad (2)$$

Equation (2) has a solution:

$$n_1(\varepsilon, t) = n_0(\varepsilon_1)(\varepsilon/\varepsilon_1)^{1/2}. \quad (3)$$

Here $n_0(\varepsilon) \equiv n(\varepsilon, 0)$ is the initial distribution, and ε and ε_1 are connected by the relation

$$\varepsilon^{3/2} = \varepsilon_1^{3/2} - 1.5\lambda_{pl}\omega_p^{5/2}t. \quad (4)$$

Equations (3, 4) describe the electronic distribution function $n_1(\varepsilon, t) \equiv n(\varepsilon, t)$ for the initial stage of the cascade. One can see directly from equation (4) that the first stage of the cascade, namely, the decrease in energy scale from ε_i to the threshold $\omega_p \ll \varepsilon_i$ is characterized by the effective time scale τ_{eff}^{pl}:

$$\tau_{eff}^{pl} \cong (\lambda_{pl}\omega_p)^{-1}(\varepsilon_i/\omega_p)^{3/2}. \quad (5)$$

Usually $\tau_{pl} \gg \tau_{eff}^{pl}$. As a result, all initial excitations with $\varepsilon \cong \varepsilon_i$ will convert into plasmons. The plasmons solution is described by the relation (see Eq. (1b))

$$N_{pl}(t) = N_0 \exp(-t/\tau_{pl}) \quad (6)$$

where

$$N_0 = \omega_p^{-1} \int_{\omega_p}^{\infty} d\varepsilon\varepsilon n_0(\varepsilon).$$

The plasmons will now decay into electron-hole excitations, and we arrive at the next stage of the cascade where the major role is played by one-particle electronic excitations with energy $\varepsilon < \omega_{pl}$. Let us turn to the description of this stage.

The region $\varepsilon < \omega_{pl}$. Electron-electron collisions

Consider now the situation of a non-equilibrium electron distribution with energies below the plasmon threshold. The one-particle excitations could be created by the plasmons decay (see above), by initial optical excitation or by electron injection so that the initial energy $\varepsilon < \omega_{pl}$. The relaxation can go through electron-electron or electron-phonon channels, and the general equation is:

$$\partial n(\varepsilon, t)/\partial t = I_{coll}^{e-e} + I_{coll}^{e-ph} \quad (7)$$

Here I_{coll}^{e-e} and I_{coll}^{e-ph} are the corresponding collision integrals. There are different optical and acoustic phonon branches: $\Omega \cong \Omega_{opt}$ and $\Omega \cong \Omega_D$, Ω is the characteristic phonon energy, Ω_{opt} is the frequency of the optical branch and Ω_D is the Debye frequency of the acoustic branch. Since $\omega_p \gg \Omega_D$, $\Omega_{opt} \gg \Omega_D$, the initial energy scale greatly exceeds the phonon frequencies. Then the major contribution comes from I_{coll}^{e-e}.

Generally speaking, the right-hand side of equation (7) also contains the term $(N_{pl}/\tau_{pl})R(\varepsilon)\theta(\omega_p - \varepsilon)$ giving the contribution of plasmon decay (N_{pl} is determined by Eq. (6), and the characteristic function $R(\varepsilon)$ is normalized: $\omega_p^{-1} \int_0^{\varepsilon_F} d\varepsilon\varepsilon R(\varepsilon) = 1$). However at $t > \tau_{pl}$ this term can be neglected.

The relaxation of an excited electron via electron-electron collisions corresponds to two processes: 1) collisions with electrons whose energy is below the Fermi level; these collisions are accompanied by creation of electron-hole pairs, and 2) collisions with other excitations at $\varepsilon > \varepsilon_F$.

A general collision integral was presented in [9,15,16]. We focus on the case 1); then $n(\varepsilon, t) \ll 1$ for $\varepsilon > \varepsilon_F$, that is, the total number of electrons greatly exceeds the number of excitations. In this case we can restrict ourselves to the linear approximation. Then we arrive at the following equation:

$$\partial n(\varepsilon, t)/\partial t = (\lambda_{ee}/\varepsilon_F)\left[- n(\varepsilon, t)\varepsilon^2/2 \right.$$
$$\left. + 3 \int_{\epsilon}^{\epsilon_F} d\varepsilon_1(\varepsilon_1 - \varepsilon)n(\varepsilon_1) \right]\theta(\varepsilon_F - \varepsilon). \quad (8)$$

The solution of this equation can be written in the following form:

$$n_2(\varepsilon, t) = 3(\lambda_{ee}/2\varepsilon_F)^{3/2}N_0\omega_p\varepsilon t^{3/2}\Gamma\left(-\frac{1}{2}, \frac{\lambda_{ee}\varepsilon^2 t}{2\varepsilon_F}\right) \quad (9)$$

where $\Gamma(\alpha, \beta)$ is an incomplete Euler gamma function, and N_0 is determined by equation (6). Equation (9) describes the electronic distribution function $n_2(\varepsilon, t) \equiv n(\varepsilon, t)$ for the regime dominated by the electron-electron collisions. One sees from equation (9) that the characteristic relaxation time is dominated by the low energy region: $\tau \propto \varepsilon^{-2}$. If final temperature is above the phonon threshold (see below), then

$$\tau_{e-e} = \left(h\epsilon_F/\lambda_{e-e}(k_BT)^2\right). \quad (9')$$

The case considered above was for $n(\varepsilon, t) \ll 1$ at $\varepsilon > \varepsilon_F$. A different situation occurs when $\varepsilon_{in}/N > E_c$, where ε_{in} is the initial energy absorbed by the system, E_c is the phonon threshold (see below, Eq. (10)), N is the total number of electrons. Such situation is realistic for systems with low concentration of electrons. In this case we are dealing with a situation when $n(\varepsilon, t) \leq 1$ for $\varepsilon > \varepsilon_F$, and collisions between the electronic excitations play a major role (contrary to the case above, when the main process involved the creation of electron-hole pairs). Moreover, since the energy region is still above the phonon threshold, we have a bottleneck scenario, i.e., slowing down of the relaxation process. This case is amendable to the two-temperature picture, because collisions between the excitations at $\varepsilon > \varepsilon_F$ can equilibrate the electronic system while maintaining the inequality $T_e \neq T_l$, where T_l is the lattice temperature.

3 Cascade. Electron-phonon collisions

3.1 General equations. The "optical phonons" stage

In the preceding section we described the initial stage of the relaxation (cascade) process when plasmon creation of and electron-electron collisions are the dominant processes. The electron energy decreases as relaxation proceeds, and finally the electron-phonon collisions become important. The intermediate region, when the contribution of both channels are similar, corresponds to the energy scale E_C determined by the relation:

$$\lambda_{ee} E_c^2/\varepsilon_F \cong \lambda\Omega. \qquad (10)$$

Subsequent decrease in energy $\varepsilon \ll E_c$ results in a the situation when the electron-phonon channel becomes dominant. Then the distribution function is determined by the equation:

$$\frac{\partial n}{\partial t} = I_{coll}^{e-ph} \qquad (11)$$

$$I_{coll}^{e-ph} = I_{coll}^{opt} + I_{coll}^{ac.} \qquad (11')$$

The first and second terms on the right-hand side of equation (11') describe collisions with optical and acoustic phonons, respectively. Specific expressions for the collision integrals can be obtained by using a method similar to that employed in our paper [5].

For acoustic phonons we obtain:

$$I_{coll}^{ac} = \left\{ -(\lambda_{ac}/3)\left(\varepsilon^3/\Omega_{ac}^2\right) n(\varepsilon, t) + 2(\lambda_{ac})/\Omega_{ac}^2 \right.$$
$$\left. \times \int_{\varepsilon}^{\Omega_{ac}} d\varepsilon_1 \varepsilon_1 (\varepsilon_1 - \varepsilon) n(\varepsilon_1, t) \right\} \theta(\Omega_{ac} - \varepsilon). \qquad (12)$$

(This expression was derived previously in [5].)

For optical phonons we obtain the following expression for I_{coll}^{opt} (see Appendix A1):

$$I_{coll}^{opt} = 4\lambda\Omega_{opt} \left\{ n(\varepsilon + \Omega, t) - n(\varepsilon, t)\theta(\varepsilon - \Omega_{opt}) \right.$$
$$\left. + (2/\Omega_{opt}) \int_{\Omega_{opt}}^{\infty} d\varepsilon_1 n(\varepsilon_1)\theta(\Omega_{opt} - \varepsilon) \right\}. \qquad (13)$$

We assume that optical (acoustic) branches are summed over.

The general equation (see Eq. (7)) can be written in the form:

$$\frac{\partial n}{\partial t} = I_{coll}^{e-e} + I_{coll}^{opt} + I_{coll}^{ac} \qquad (7')$$

where the terms on the right-hand side are determined by equations (8, 12) and (13), respectively.

The phonon spectra of simple metals do not contain optical modes in which case one can focus directly on I_{coll}^{ac}. However, the situation for complex compounds is entirely different; here optical modes are playing an important role. Usually $\Omega_{opt} > \Omega_{ac}$. For such systems, it is natural to consider, at first, a case when the distribution function $n(\varepsilon, t)$ is described by equation (11) with $I_{coll} = I_{coll}^{opt}$, see equation (13).

Let us evaluate the characteristic time $\tau_{opt.}$ describing this stage of the cascade. We introduce the quantities $\phi_K(t)$ defined by the relation:

$$\phi_K(t) = \int_{(K+1)\Omega}^{\infty} d\varepsilon\, n(\varepsilon, t), \quad K = 0, 1, 2, \dots \qquad (14)$$

It is easy to see that these functions satisfy equations:

$$\partial\phi_K/\partial t = -\mu(\phi_K - \phi_{K+1}) \qquad (14')$$

where

$$\mu = 4\lambda\Omega.$$

The solution of equation (14) can be written in the form:

$$\phi_K(t) = \exp(-\mu(t - t_0)) \sum_{m=0}^{\infty} \phi_{K+m}(0)(\mu(t - t_0))^m/m!. \qquad (15)$$

Here t_0 is determined by the relation

$$\lambda_{ee} E_c^2 t_0/2\varepsilon_F = 1 \qquad (16)$$

which follows directly from equation (9); $\phi_K(0) \equiv \phi_K(t_0)$.

The functions $\phi_K(0)$ are continuous functions of the parameter K, and for $K < E_C\Omega^{-1}$ one can write:

$$\phi_K(0) = \phi_0(0)(1 - \gamma K), \qquad (17)$$

where

$$\phi_0(0) = 3\sqrt{\pi}N_0\omega_p/2E_C; \quad \gamma = 4\Omega/\sqrt{\pi}E_C.$$

With the use of these relations and equation (15), we obtain:

$$\phi_0(t) = \phi_0(0)\left\{ 1 - 4\mu(t - t_0)\Omega/\sqrt{\pi}E_C \right\}. \qquad (15')$$

This relation is valid for the range $0 < t - t_0 < \tau_{opt}^*$, with the characteristic time

$$\tau_{opt} = \sqrt{\pi} E_C / 8\lambda\Omega^2. \qquad (18)$$

The expression (18) determines the time scale for the cascade stage dominated by the electron-optical phonons scattering (E_c is determined by Eq. (10)).

Let us evaluate the electronic distribution function $n(\varepsilon, t)$ corresponding to this stage of the cascade. The initial function $n(\varepsilon, t_0)$ can be written as $n_2(t_0)$ (see Eq. (9)). The function of interest $n_3(\varepsilon, t)$ can be sought in the form:

$$n_3(\varepsilon, t) = (3N_0\omega_{pl}/\Omega E_C)(\Omega/E_C)^2 \exp[-\mu(t - t_0)]$$
$$\times \sum_{m=0}^{\infty} [(\varepsilon/\Omega) + m][\mu(t - t_0)]^m (m!)^{-1}$$
$$\times \Gamma\left[-\frac{1}{2}, \left(\frac{\varepsilon}{E_C} + \frac{m\Omega}{E_C}\right)^2\right]. \qquad (19)$$

Since we are interested in $t - t_0 \gg \mu^{-1}$, the main contribution to the sum comes from the region $m \gg 1$. Then the summation can be replaced by integration, and we obtain the following analytical expression for the distribution function:

$$n_3(\varepsilon, t) \cong \frac{3N_0\omega_{pl}}{\Omega E_C}\left(\frac{\Omega}{E_C}\right)^2 [(\epsilon/\Omega) + 4\lambda\Omega(t - t_0)]$$
$$\times \Gamma\left[-\frac{1}{2}, \left(\frac{\varepsilon}{E_C} + \frac{4\lambda\Omega^2(t - t_0)}{E_C}\right)\right]. \qquad (20)$$

Equation (20) describes the electronic distribution function $n_3(\varepsilon, t) \equiv n(\varepsilon, t)$ for that part of the cascade where the scattering of electrons by optical phonons is dominant.

3.2 Relaxation via acoustic phonons

For many systems the final stage of the cascade involves collisions of the carriers with acoustic phonons (see below for a discussion of some special cases). This channel of relaxation was discussed by us in [5]. Here for completeness of a whole scenario and to for some additional insight we describe a more general derivation.

The equation for the distribution function has the following form:

$$\partial n(\varepsilon, t)/\partial t = 8\lambda\phi_0(t) + \Big\{ -(\lambda_{ac}/3)\left(\varepsilon^3/\Omega_{a.p.}^2\right)n(\varepsilon, t)$$
$$+ 2\left(\lambda_{ac}/\Omega_{a.p.}^2\right)\int_\varepsilon^{\Omega_{a.p.}} d\varepsilon_1\varepsilon_1(\varepsilon_1 - \varepsilon)n(\varepsilon, t)\Big\}\theta(\Omega_{a.p.} - \varepsilon). \qquad (21)$$

The distribution function can be sought in the scaling form

$$n(\varepsilon, t) = \mu^2(t)U(\varepsilon\mu(t)) \qquad (22)$$

where

$$\mu(t) = [3C_1(t + t_1)]^{1/3}. \qquad (23)$$

Here C_1, t_1 are some constants (see [1]). In [1] we used a step-like approximation for U. More complicated expression was used in [18], see below. It is worth noting that because of the scaling form, the important and non-trivial dependence $\tilde{n}(t)$, where $\tilde{n}$ is the total number of excitations (see below, Eq. (27)) does not depend on the exact form of the U-factor.

With the use of equations (21–23) we arrive, after some manipulations, at the following expression:

$$U = B\Gamma(1/3; \quad \lambda_{a.p.}(t + t_1)\varepsilon^3/3\Omega_{a.p.}^2. \qquad (24)$$

Note that the more complicated expression obtained in [18] can be reduced to (22–24) (see Appendix A2).

As mentioned, usually $\Omega_{opt} > \Omega_D$, so that initially the electron-phonon channel is dominated by electron-optical phonon scattering. If the optical branches are absent (e.g., for simple monoatomic metals), then at $\varepsilon < E_c$ the scattering by phonons becomes essential (it is assumed that $T < E_c$). One can see from equation (24) that the characteristic time τ^{ac} is determined by the following expression:

$$\tau^{ac} = 3h(\Theta_D/T)^2(\lambda_{e-p}k_BT)^{-1}. \qquad (25)$$

Using equations (22, 24), one can evaluate the total number of excitations $\tilde{n}(t)$ during this stage and we obtain the dependence obtained by us in [5]

$$\tilde{n}(t) = n_0[1 + \alpha t]^{1/3} \qquad (26)$$

where $\alpha = (\lambda_{a.p.}/2)\Omega_D$. This dependence was subsequently also obtained in [17].

For the sample with dimensions L, there is a characteristic temperature $T' \approx hu/k_BL$, so that at $T < T'$ the phonon wavelength exceeds L. This frustrates the electron-phonon interaction and the relaxation process at $T < T'$, leading to the appearance of a low temperature bottleneck. For example, for Ag with $L \approx 45$ nm and for Au with $L \approx 30$ nm [11]; then $T' \approx 20$ K. In such a case, electron-electron collisions again lead to the two temperatures picture.

A special case corresponds to situation when the material in question is a superconductor. This case was described by us in [5]. Here relaxation is governed by the electron-phonon scattering for $\varepsilon > 2\Delta$, where Δ is the pairing energy gap. At lower energies, Cooper pair recombination becomes the dominant process. As we have mentioned, this situation and its application to superconducting tunneling detectors were described in our previous paper [5].

4 Experimental data. Discussion

The number of cascade stages for each specific practical case is determined by the energy of initial excitations and by the temperature of the environment. The most general

case corresponds to the initial energy exceeding the plasmon energy and measurements being performed at low temperature, near $T = 0$ K. In this case the cascade will go through all channels described above (see Sects. 2, 3). On the other hand, if $T > \Theta_{opt.}, \Theta_D$ then the equilibrium which corresponds to temperature T is established, mainly, through the electronic channels. The relaxation time corresponding to electron-electron collisions is determined by equation (9); as was noted above, τ_{e-e} is determined, mainly, by the low energy region. If $T > \Theta_{opt.}, \Theta_D$, then $\tau \approx \tau_{e-e}$ (see Eq. (9')).

The measurements of relaxation for the Ag and Au films are described in [11]. The experiments were performed at room temperature. For such case $T > \Omega_D$ ($\Omega_D^{Ag} \approx \Omega_D^{Au} \approx 200$ K), and, therefore, one can use equation (9') in order to estimate the relaxation time. The value of the Fermi energy for the films are $\varepsilon_F \approx 6 \times 10^4$ K. The relaxation time depends also on the value of the electronic constant λ_{e-e}. Its value can be obtained from the tunneling spectroscopy (see, e.g., [18,19]). Indeed, the tunneling spectroscopy which is usually performed for superconducting metals, allows to determine the Coulomb pseudopotential μ^* defined by the relation:

$$\mu^* = \lambda_{e-e}/[1 + \ln(\varepsilon_F/\Omega)]. \qquad (27)$$

Here $\lambda_{e-e} = U_{e-e}\nu$ is the electron-electron constant, U_{e-e} is the matrix element, and ν is the density of states, Ω is the characteristic phonon frequency (for simple metals $\Omega \approx \Omega_D$). As is known, the value of the $\mu*$ is about 0.15 for noble metals. Note, by the way, that the value of the electron-phonon coupling for these materials is $\lambda_{e-p} = 0.17$ (see [19]). As a result, the quantity λ_{e-p}-λ_{e-e} is very small and this leads to these metals being normal, not superconducting. Using this value $\mu^* = 0.15$, we obtain from equation (27) the value of $\lambda_{e-e} = 1.5$ for Ag, Au. Using it together with ε_F (see above), we obtain, employing equation (9'), a relaxation time $\tau = 3$ ps, in agreement with measurements in [11].

Another interesting case is relaxation via electron-electron scattering in n-type GaAs. Such a process was observed at room temperature in [20]. The sample was a highly diluted semiconductor with a degenerate electronic system and carrier concentration $n = 10^{18}$ cm^{-3}. The analysis can be based on equation (9'). The value of the parameter λ_{e-e} for diluted Ga As can be estimated from the relation $\lambda_{e-e} = U_{e-e}\nu$, where $U_{e-e} = 4\pi e^2[\varepsilon_b q^2 + k_s^2]^{-1}$ is the matrix element (see, e.g., [13,21]), and $\nu = m^* k_F/\pi h^2$ is the density of states. Here ε_B is the background dielectric constant, q is the transferred momentum ($q \approx k_F$), and $k_s = 0.8 r_s k_F$ is the screening parameter; $r_s = (3/4\pi n)^{-1/3} a_B$, a_B is the Bohr radius. For GaAs $\varepsilon \approx 10$, $m^* \approx 7 \times 10^{-2}$; in addition, $k_F = (3\pi^2 n)^{1/3}$ for $n = 10^{18}$. Using these values, we obtain $\tau \approx 50$ fs, in agreement with the data [20].

As noted above, electron-electron collisions make the main contribution to the relaxation time if $T > \Theta_{opt.}, \Theta_D$. In the opposite case of low temperatures measurements, so that $T < \Theta_D$, the electron-acoustic phonon channel is important. This channel was described in detail in our previous paper [5] and good agreement was found between the data [23,24] and the results of the theory (Eq. (26)).

Let us discuss also relaxation in tungsten studied experimentally in [28]. This case is interesting, because W is characterized by relatively high value of the Debay temperature ($\Theta_D \approx 400$ K, see, e.g., [22]) and, in addition, by relatively large value of $\lambda_{e-p} \approx 0.3$ [18]. As a result, relaxation time is mainly determined by electron-phonon channel (Eq. (25)). Since $T \approx 3 \times 10^2$ K, we obtain with use of equation (25) $\tau \approx \tau^{ac} \approx 4 \times 10^2$ fs, in agreement with the measurements [8].

In summary, we have developed a microscopic description of the relaxation cascade, a strong non-stationary phenomenon. Generally speaking, the relaxation process consists of several stages. It is initially dominated by electronic channels (creation of plasmons followed by electron-electron scattering), and at lower energies electron-phonon scattering plays a key role. We have evaluated the electronic distribution function $n(\varepsilon, t)$ for the entire range of energies (Eqs. (3, 9, 20), and (24)) and the characteristic relaxation times (Eqs. (5, 9, 18), and (25)) for various channels.

Appendix

A1

The derivation of equation (13) is based on the general expression for the collision integral obtained by A. Larkin and one of the authors [25,26] (see also our paper [5]):

$$I^{ph} = \frac{i\lambda}{4\pi} \int d\Omega_{p_1} \int \frac{d\varepsilon_1}{2\pi} \text{Tr}\left\{\hat{\delta}_{\bar{p}}(\varepsilon)\hat{\delta}_{\bar{p}_1}(\varepsilon_1)\left[\hat{\tilde{D}}_{\bar{p}-\bar{p}_1}(\varepsilon_1 - \varepsilon)\right.\right.$$
$$\times (f_{\bar{p}}(\varepsilon) - f_{\bar{p}_1}(\varepsilon_1)) + \left(D_{\bar{p}-\bar{p}_1}^R(\varepsilon_1 - \varepsilon) - D_{\bar{p}-\bar{p}_1}^A(\varepsilon_1 - \varepsilon)\right)$$
$$\left.\left.\times (1 - f_{\bar{p}}(\varepsilon)f_{\bar{p}_1}(\varepsilon_1))\right]\right\}. \quad (A.1)$$

Here λ is the electron-phonon coupling constant, $\delta_p = (\hat{g}_p^R - \hat{g}_p^A)/2$, $\hat{g}_p^{A(R)}$ are the advanced (retarded) electronic Green's functions integrated over energy [27,28], $f = 1 - 2n(\varepsilon, t)$, where $n(\varepsilon, t)$ is the quasiparticle distribution function, and $D^{R(A)}$ is the retarded (advanced) phonon Green's function:

$$D_k^R(\omega) = D_k^{A*}(\omega) = -\Omega^2\left(\bar{k}\right)\left[\Omega^2\left(\bar{k}\right) - (\omega + i\delta)^2\right]^{-1} \quad (A.1')$$

into which one can substitute an appropriate dispersion relation for the optical or acoustic phonons. In addition,

$$\hat{\tilde{D}}_{\bar{p}}(\omega) = \left(D_p^R - D_p^A\right)(1 + 2N_{ph}(|\omega|))\text{sgn }\omega \quad (A.1'')$$

where N is the phonon distribution function.

With the use of these expressions and equation (A.1), we obtain (*cf.* [1]):

$$\partial n(\varepsilon)/\partial t = 4\lambda\Omega\{-n(\varepsilon)\theta(\varepsilon - \Omega)$$
$$+ n(\varepsilon + \Omega) + N_{ph}(\Omega - \varepsilon)\} \tag{A.2}$$

$$\partial N_{ph}/\partial t = 4\lambda\Omega/\Lambda\left\{2\int_{\Omega}^{\infty}\mathrm{d}\varepsilon n(\varepsilon) - \Omega N_{ph}\right\}. \tag{A.2'}$$

Here Λ is the width of the phonon band. In the first approximation we can neglect the derivative $\partial N_{ph}/\partial t$ (*cf.* [5]). Excluding N_{ph} from equations (A.1, A.2), we arrive at equation (13).

A2

Equation (21) can also be analyzed by a different method. It can be rewritten as follows:

$$\partial^2 z/\partial t\partial\varepsilon = -2\lambda_{a.p.}(\varepsilon/\Omega_{a.p.})^2 z - \lambda_{a.p.}\left(\varepsilon^3/3\Omega_{a.p.}^2\right)\partial z/\partial\varepsilon \tag{A.3}$$

where

$$z = \partial n(\varepsilon, t)/\partial\varepsilon.$$

The solution of equation (A.3) can be sought in the following form:

$$z = \exp\left[-\lambda_{a.p.}t\varepsilon^3/3\Omega_{a.p.}^2\right]\phi(\varepsilon, t). \tag{A.4}$$

With the use of equations (A.3, A.4), we obtain:

$$\partial^2\phi/\partial\varepsilon\partial t - \tilde{\lambda}(t\partial\phi/\partial t - \phi) = 0 \tag{A.5}$$
$$\tilde{\lambda}_{a.p.} = \lambda_{a.p.}\varepsilon^2/\Omega_{a.p.}^2.$$

The solution of this equation has the form:

$$\phi(\varepsilon, t) = \phi_1(\varepsilon) + t\phi_2(\varepsilon). \tag{A.5'}$$

Here $\phi_1(\varepsilon) = \frac{\partial n_0(\varepsilon)}{\partial\varepsilon}$, and ϕ_2 is determined by the relation $\partial\phi_2/\partial\varepsilon = -\tilde{\lambda}_{a.p.}\phi_1$. As a result we obtain the following expression:

$$n(\varepsilon, t) = n_0(\varepsilon)\exp\left(-\lambda_{a.p.}t\varepsilon^3/3\Omega_{a.p.}^2\right)$$
$$+ 2\lambda_{a.p.}t/\Omega_{a.p.}^2\int_{\epsilon}^{\infty}\mathrm{d}\varepsilon_1\exp\left(-\lambda_{a.p.}t\varepsilon_1^3/3\Omega_{a.p.}^2\right)\mu(\varepsilon_1) \tag{A.6}$$

$$\text{where} \quad \mu(\varepsilon_1) = \int_{\varepsilon_1}^{\infty}\mathrm{d}\varepsilon_2\varepsilon_2 n_0(\varepsilon_2). \tag{A.6'}$$

Equation (A.6) was previously obtained in [17]. We can put $\varepsilon_1 = 0$ in (A.6'), and after a short interval, the first term will become small and we arrive at expressions (22, 24).

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